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Implications of neutrino masses on the $K_L \to \pi^0 \nu\bar{\nu}$ decay

Gilad Perez

Department of Particle Physics,
Weizmann Institute of Science, Rehovot 76100, Israel
E-mail: perez@clever.weizmann.ac.il

Abstract: We calculate the different contributions to the decay $K_L \to \pi^0 \nu\bar{\nu}$ that arise if the neutrinos are massive. In spite of a chiral enhancement factor, we find that these contributions are negligibly small. Compared to the CP violating leading contributions, the CP conserving contributions related to Dirac masses are suppressed by a factor of order $(m_K m_\nu / m_W^2)^2 \lesssim 10^{-12}$, and those related to Majorana masses are suppressed by a factor of order $(\alpha_W m_K m_s^2 m_\nu / m_W^4)^2 \lesssim 10^{-29}$. With lepton flavor mixing we find new contributions with a single CP violating coupling leading to a final CP even state or with two CP violating couplings leading to a final CP odd state. These contributions can be of the order of the Standard Model CP conserving contributions to the flavor diagonal modes.

Keywords: CP violation, Neutrino Physics, Rare Decays, Kaon Physics.

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Contents

1. Introduction

The decay $K_L \to \pi^0 \nu\bar{\nu}$ is known to be a purely CP violating (CPV) process to a very good approximation [1] and subject to a clean theoretical interpretation. Within the standard model (SM), it will determine the physical CPV phase (η) [2]. Beyond the SM, it will probe new sources of CPV [3].

The general argument that the CP conserving (CPC) contributions are small is based on the chiral expansion [4]. Within the SM, these contributions have been calculated and found to be indeed negligible [5].

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If, however, neutrinos were massive then a scalar four fermion operator would be allowed. Such an operator is CPC and would be the lowest order in the chiral expansion. On the other hand, it is suppressed by the small neutrino mass. In addition, if neutrinos are massive then flavor mixing is possible. A $\pi^0\nu_i\bar{\nu}_j$ final state with $i \neq j$, which is not a CP eigenstate, is allowed [3]. This would give rise to additional contributions with a single or double CPV coupling leading to a CP even or CP odd final states respectively. In this work we calculate the different contributions related to neutrino masses and mixing and find whether they can be significant.

The models that we consider here are minimal extensions of the SM. We take the SM lagrangean and add to it the Yukawa interactions that are necessary to induce neutrino masses. We assume that all other effects of new physics related to neutrinos are negligibly small and do not affect the $K_L \to \pi^0\nu\bar{\nu}$ decay. We consider two types of neutrino masses:

1. Dirac masses, which require the existence of right handed neutrinos. Lepton number is conserved but there is no understanding of why neutrinos are lighter than charged fermions. We assume that the right-handed neutrinos are only involved in Yukawa interactions. In particular, if they have gauge interactions beyond the SM, we assume that these interactions take place at a high enough energy scale that they effectively decouple.

2. Majorana masses, which require that the SM is taken to be a low energy effective theory only. Lepton number is violated and there is a natural explanation of the lightness of neutrinos [6]. We assume that the scale of lepton number violation is high enough that any other effects of physics at that scale effectively decouple.

We study Dirac neutrinos in section 2 and Majorana neutrinos in section 3. The effects of lepton flavor mixing are analyzed in section 4. A summary of our conclusions is given in section 5.

2. Massive Dirac neutrinos

In the presence of neutrino Dirac masses, new CPC operators could appear in the effective hamiltonian: a scalar operator,

$$\mathcal{H}_{eff}^{scal} = (\bar{s}d)(\bar{\nu}\nu)\,, \tag{2.1}$$

and a tensor operator, $\mathcal{H}_{eff}^{ten} = (\bar{s}\sigma^{\mu\nu}\gamma^5 d)(\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu)$. The tensor operator contribution is expected to be smaller than the scalar one [7], and we do not consider it any further here.

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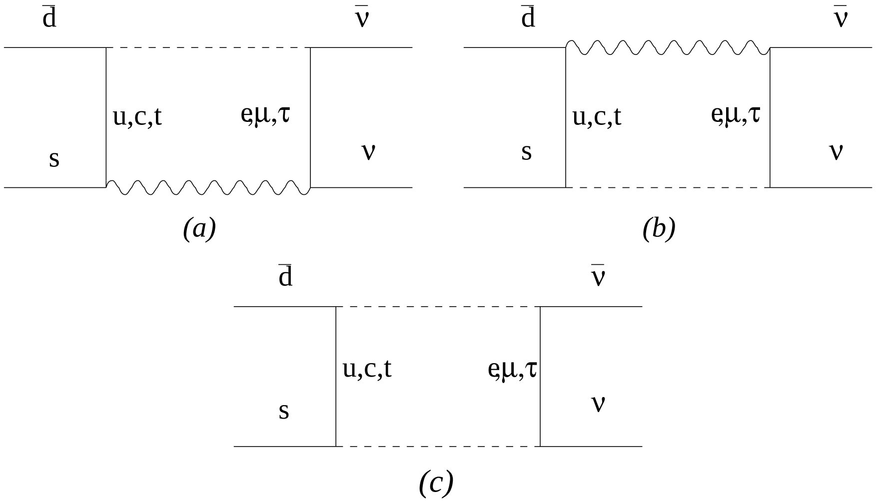

Figure 1: The Standard Model box diagrams.

2.1 The model

We consider an extension of the SM where we add three right-handed (singlet) neutrinos and impose lepton number conservation. The right-handed neutrinos have no gauge interactions, but they have Yukawa couplings:

$$\mathcal{L}_Y = f_{mn} \overline{L^m_{Li}} \nu^n_R \tau^{ij}_2 \Phi^*_j + \text{h.c.}\,, \tag{2.2}$$

where L_L is a left-handed lepton doublet, Φ is the Higgs doublet and ν_R is a right-handed neutrino singlet; m, n are generation indices and i, j are $\mathrm{SU}(2)$ ones. Similarly to the SM, the only source of flavor changing couplings (in the mass basis) are the charged current interactions, while the Higgs couplings are diagonal. In this section we consider only outgoing neutrinos which carry the same flavor. CPV and mixing effects in the lepton sector will be treated separately in section 4.

2.2 The decay rate calculation

We calculate the amplitude in the full theory and without QCD corrections. Neglecting the long distance contributions is justified since we find that the dominant contribution arises from loop momenta $k \sim M_W$. The omission of QCD corrections does not affect our conclusions.

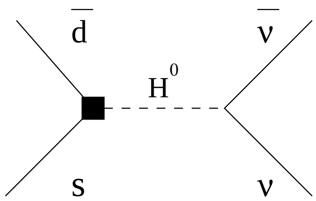

Figure 2: The Higgs-mediated diagram for the case of Dirac masses.

The diagrams that generate the scalar operator (2.1) are of two types. First, there are the SM box diagrams drawn in fig. 1. Second, there is a neutral Higgs mediated penguin diagram, related to the Yukawa interaction of eq. (2.2). Replacing the SM loop contribution with an effective sdH coupling [8], we can consider it to be an "effective tree" diagram drawn in fig. 2.

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2.2.1 The box diagram contributions

In contrast to the calculation of the CPV contributions from box diagrams, external momenta should not be neglected in our calculation. The external momentum expansion will produce a CPC operator of the form $\overline{d}p_s^\mu\gamma_\mu s \sim \overline{d}m_s s$, which is comparable with the other contributions. The calculation is, however, simplified by the fact that the dominant terms have at most a linear dependence on the external momentum. Contributions suppressed by a factor of order $(m_i m_j)/M_W^2$ (where $i,j = u,d,\nu$) are subdominant and we neglect them.

Summing the dominant contributions from the box diagrams (fig. 1), we get:

$$\mathcal{H}^1_{CPC} = \sum_{i,k,\ell} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\Theta_W} \lambda_i \widetilde{V}^*_{\ell k} \widetilde{V}_{\ell k} (\bar{d}s)(\overline{\nu}_\ell \nu_\ell) X_B(x_i) + \text{h.c.}\,, \tag{2.3}$$

with i,k,ℓ the flavor indices of the internal quark, internal lepton and external neutrinos respectively; $\widetilde{V}$ is the MNS lepton mixing matrix [9];

$$X_B(x_i) = \frac{m_\nu m_s}{2M_W^2} \left\{ \frac{(-x_i + 1 + x_i \ln(x_i))}{(x_i-1)^2} - \frac{1}{2}\frac{x_i(-2x_i + \ln(x_i)x_i + \ln(x_i) + 2)}{(-1+x_i)^3} \right\} \tag{2.4}$$

and $x_i = (m_i/M_W)^2$. Since the top quark contribution is dominant, (2.4) can be simplified:

$$\sum_i \lambda_i X_B(x_i) \approx \lambda_t \frac{m_\nu m_s}{M_W^2} \frac{x_t}{2(x_t-1)^2} \left(2 - x_t + \ln(x_t)\frac{x_t - 3}{2(x_t-1)}\right). \tag{2.5}$$

Moreover since X_B (2.4) is to leading order m_ℓ independent and by the virtue of $\widetilde{V}$ unitarity, the sum over k is trivial:

$$\sum_k \widetilde{V}^*_{\ell k}\widetilde{V}_{\ell k} X_B(x_i) = X_B(x_i)\,. \tag{2.6}$$

We learn that CPC contributions from box diagrams are suppressed by a factor of $\mathcal{O}(m_\nu m_s/M_W^2)$.

2.2.2 The Higgs-mediated contribution

We calculate the Higgs-mediated diagram of fig. 2. The effective $s\overline{d}H$ coupling Γ arising from the SM loop (represented as a square in fig. 2) is given by [8]:

$$\Gamma \equiv -\lambda_t \frac{g^3}{128\pi^2}\frac{m_t^2 m_s}{M_W^3}\left(\frac{3}{2} + \frac{m_H^2}{M_W^2} f_2(x_t)\right)(1+\gamma^5)\,, \tag{2.7}$$

where

$$f_2(x) \equiv \frac{x}{2(1-x)^2}\left(-\frac{x}{1-x}\ln x + \frac{2}{1-x}\ln x - \frac{1}{2} - \frac{3}{2x}\right). \tag{2.8}$$

Since the coupling in eq. (2.7) is by itself proportional to m_s/M_W, we can safely neglect the external momentum. Thus the amplitude of fig. 2 is given by:

$$i\mathcal{T}^2 = -\sum_\ell \frac{G_F}{\sqrt{2}} \frac{\alpha}{(2\pi)\sin^2\Theta_W} \frac{i}{2}\frac{m_t^2 m_s m_\nu}{M_H^2 M_W^2}\lambda_t \left(\frac{3}{2} + \frac{m_H^2}{M_W^2} f_2(x_t)\right)(\bar{d}s_R)(\overline{\nu}_\ell\nu_\ell)\,, \tag{2.9}$$

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2.2.3 The rate

Adding the contributions of the Higgs-penguin and the box diagrams (eqs. (2.3) and (2.9)), we get:

$$\mathcal{H}_{CPC}^{Dir} = \sum_{\ell} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \lambda_t \ (\bar{d}s)(\overline{\nu}_\ell \nu_\ell) X^{Dir} + \text{h.c.}\,, \tag{2.10}$$

where

$$X^{Dir} \approx \frac{m_\nu m_s}{M_W^2} \frac{x_t}{2} \left[\frac{1}{(x_t-1)^2} \left(2 - x_t + \ln(x_t) \frac{x_t - 3}{2(x_t-1)} \right) - \frac{1}{2} \frac{M_W^2}{M_H^2} \left(\frac{3}{2} + \frac{m_H^2}{M_W^2} f_2(x_t) \right) \right]. \tag{2.11}$$

The ratio between the scalar hadronic matrix element and the $V-A$ hadronic matrix element (related to the leading CPV contribution) is found via the use of the equation of motion [10]:

$$\frac{|\ \langle \pi^0 | \bar{s} d | K^0 \rangle\ |}{|\ \langle \pi^0 | (\bar{s} d)_{V-A} | K^0 \rangle\ |} \sim \frac{M_K}{m_s} \approx 3\,. \tag{2.12}$$

Using eqs. (2.10) and (2.12), we can now compare the leading CPV rate [11] to the CPC one with massive Dirac neutrinos:

$$R_{CPV}^{Dir} = \frac{\Gamma_{CPC}^{Dir}(K_L \to \pi^0 \nu \bar{\nu})}{\Gamma_{CPV}(K_L \to \pi^0 \nu \bar{\nu})} \sim \left(\frac{M_K}{M_W} \frac{m_\nu}{M_W} \right)^2 \approx 10^{-12} \left[\frac{m_\nu}{10 \text{ MeV}} \right]^2. \tag{2.13}$$

The direct experimental upper bound [12], $m_{\nu_\tau} \leq 18.2$ MeV, implies that indeed the CPC contribution from neutrino Dirac masses is suppressed by at least twelve orders of magnitude compared to the CPV contribution of the SM. However, a significantly lower upper bound ($m_\nu \lesssim O(10$ eV$)$) is found from astrophysical and cosmological reasoning [13], so that very likely $R_{CPV}^{Dir} \lesssim 10^{-24}$. We learn that the addition of a small Dirac mass to the SM neutrinos does not change the statement that the $(K_L \to \pi^0 \nu \bar{\nu})$ decay is purely CPV to an excellent approximation.

3. Massive Majorana neutrinos

In the presence of neutrino Majorana masses, a new scalar, CPC operator appears in the effective hamiltonian:

$$\mathcal{H}_{eff}^{Maj} = (\bar{s}d)(\nu\nu). \tag{3.1}$$

3.1 The model

We consider the SM as a low energy effective theory, that is we allow for non-renormalizable terms. Neutrino Majorana masses are induced via the following dimension-five terms:

$$\mathcal{L}_{\nu\nu} = \frac{1}{2} \left(\frac{f_{mn}}{M} \right) (L_{Li}^m)_\alpha \sigma_{\alpha\beta}^2 \tau_{ir}^2 \tau_{rj}^t (L_{Lj}^n)_\beta \Phi_k \tau_{ks}^2 \tau_{sl}^t \Phi_l + \text{h.c.}\ , \tag{3.2}$$

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where m, n the generation indices, i, j, k, l, r, s, t are SU(2) indices and α, β are spinor indices. In this section we assume again flavor diagonality. Flavor mixing is discussed in the next section.

The interaction term (3.2) induces neutrino masses, $(M_\nu^{Maj})_{mn} \sim f_{mn} \frac{\langle \Phi^0 \rangle^2}{M}$. In addition, it generates a new CPC contribution to the $K_L \to \pi^0 \nu \bar{\nu}$ decay. The diagram which leads to the CPC contribution is shown in fig. 3. The effective $\nu_L^T \sigma^2 \nu_L HH$ coupling of the diagram in fig. 3 can be written in terms of the neutrino mass:

$$-\frac{f_{ii}}{M} = -g^2 \frac{m_{\nu_i}^{Maj}}{2M_W^2}, \tag{3.3}$$

with $i = 1, 2, 3$. Note that the diagram includes the effective $s\bar{d}H$ coupling given in eq. (2.7).

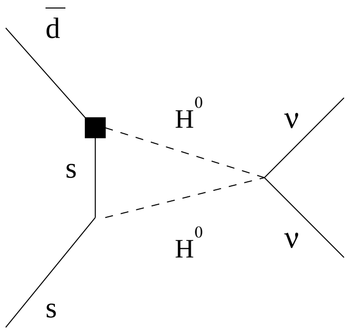

Figure 3: The Majorana case CPC diagrams.

3.2 The decay rate calculation

3.2.1 The amplitude

The diagram in fig. 3 is calculated using standard techniques. The fact that we have two suppressed vertices allows us to neglect the external momenta. We again neglect QCD corrections. The short distance contribution is:

$$\begin{aligned} i\mathcal{T}^3 = &-\sum_{i,k,\ell} \frac{G_F}{\sqrt{2}} \frac{\alpha}{(2\pi)\sin^2\Theta_W} (M_W^2 16\pi^2) \lambda_t \frac{m_t^2 m_s^2 m_\nu^{Maj}}{M_W^6} \frac{g^2}{128\pi^2} \left(\frac{3}{2} + \frac{m_H^2}{M_W^2} f_2(x_t)\right) \times \\ &\times \int_\mu^\infty \frac{d^4k}{(2\pi)^4} \frac{i}{(k^2 - M_H^2)^2} \left[\bar{d}_R i \frac{\gamma \cdot k + m_s}{k^2 - m_s^2} s\right] \left((\nu_{\ell L})^T \sigma^2 \nu_{\ell L}\right). \end{aligned} \tag{3.4}$$

Thus we get the following CPC operator:

$$\mathcal{H}_{CPC1}^3 \approx \sum_{i,\ell} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\Theta_W} \lambda_i X_{Maj1} \; (\bar{d}s_R) \left((\nu_{\ell L})^T \sigma^2 \nu_{\ell L}\right) + \text{h.c.}, \tag{3.5}$$

where

$$X_{Maj1} = \frac{1}{4} I_{Maj1} g^2 \frac{m_t^2 m_s^3 m_\nu^{Maj}}{M_H^2 M_W^4} \left(\frac{3}{2} + \frac{m_H^2}{M_W^2} f_2(x_t)\right) \tag{3.6}$$

and, defining $y_s \equiv m_s^2 / M_H^2$,

$$I_{Maj1} = \int_{\mu/M_H}^\infty \frac{-id^4k}{(2\pi)^4} \frac{1}{(k^2-1)^2} \frac{1}{k^2 - y_s} \approx \frac{1 + 2y_s \ln(\mu/M_H)}{32\pi^2}. \tag{3.7}$$

Note that the long distance contributions (namely the contributions from $k \lesssim \Lambda_{QCD}$ in 3.7), are logarithmic and suppressed by $y_s \sim 10^{-6}$. Therefore they are subdominant and neglected.

The hadronic matrix element is the same as in the case of the Dirac neutrinos. Its ratio with the leading CPV hadronic operator was presented above (2.12).

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3.2.2 The rate

We can now compare the Majorana mass contribution to the CPC rate with the leading CPV [11] one. Using (2.12) and (3.5), we find:

$$R^{Maj}_{CPV} \equiv \frac{\Gamma^{Maj}_{CPC}(K_L \to \pi^0\nu\bar\nu)}{\Gamma^{SM}_{CPV}(K_L \to \pi^0\nu\bar\nu)} \sim \left(\frac{3\alpha}{64\pi\sin^2\Theta_W}\frac{m_t^2}{M_H^2}\frac{m_s^3}{M_W^3}\frac{m_\nu^{Maj}}{M_W}\frac{M_K}{m_s}\right)^2$$
$$\approx 10^{-29}\left[\frac{m_\nu}{10\text{ MeV}}\right]^2\left[\frac{100\text{ GeV}}{M_H}\right]^4. \tag{3.8}$$

We learn that the CPC contribution from neutrino Majorana masses is suppressed by at least twenty nine orders of magnitude compared to the CPV contribution of the SM. Since, very likely, $m_{\nu_\tau} \lesssim 10$ eV [13], we expect $R^{Maj}_{CPV} \lesssim 10^{-41}$.

The strong suppression of the massive Majorana CPC amplitude is a result of four factors: first, the CPC operator $\nu_L^T\sigma^2\nu_L$ is created via the $\nu_L^T\sigma^2\nu_L HH$ vertex. The vertex carries suppression factor m_ν^{Maj}/M_W. A second factor, $\frac{3g^2}{128\pi^2}\frac{m_s}{M_W}$, arises from the flavor diagonality of the tree level Yukawa coupling: it is the loop suppression of the $s\bar{d}H$ coupling. The third factor, m_s/M_W, arises from the $s\bar{s}H$ Yukawa coupling. The fourth one, m_s/M_W, arises due to the required helicity flip along the internal s quark propagator.

4. Lepton flavor mixing

In the presence of masses, the neutrino mass basis is in general not equal to the interaction basis, leading to lepton flavor mixing. The effects of mixing on the different contribution to the $K_L \to \pi^0\nu\bar\nu$ decay are analyzed below. We first investigate the effects of mixing on the contributions related to the scalar operator. Then we point out that the contributions related to $V-A$ operators can lead, through two insertions of CP violating vertices, to CP odd final states.

4.1 Scalar operators

4.1.1 Dirac neutrinos

With flavor mixing, the effective hamiltonian which governs the decay acquires the following form:

$$\mathcal{H}^{Dir}_{mix} = \sum_{j,k,\ell}\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}\lambda_t(\bar{d}s)(\overline{\nu}_j\nu_k)\widetilde{V}^*_{j\ell}\widetilde{V}_{k\ell}X^{Dir}(x_t,x_\ell) + \text{h.c.}\,, \tag{4.1}$$
$$X^{Dir}(x_t,x_\ell) = \frac{m_\nu m_s}{M_W^2}\frac{x_t}{2}\left[\frac{1}{(x_t-1)^2}\left(2 - x_t + \ln(x_t)\frac{x_t-3}{2(x_t-1)}\right) + \frac{3}{16}\frac{M_W^2}{M_H^2} + O(x_\ell)\right].$$

The unitarity of $\widetilde{V}$ and the smallness of the charged lepton masses, $x_\ell < 10^{-3}$, induce a very effective leptonic GIM mechanism. Consequently, to a very good

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approximation, $X^{Dir}(x_t, x_\ell)$ is independent of x_ℓ and the sum over ℓ in (4.1) gives:

$$\mathcal{H}^{Dir}_{mix} \cong \sum_{\ell} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \lambda_t \, (\bar{d}s)(\overline{\nu}_\ell \nu_\ell) X^{Dir}(x_t) + \text{h.c.} \,. \tag{4.2}$$

This is nothing but $\mathcal{H}^{Dir}_{CPC}$ of eq. (2.10) which was derived without mixing. Thus the effects of lepton flavor mixing here are negligible.

4.1.2 Majorana neutrinos

The new contribution to the decay is generated by the $\nu\nu HH$ vertex. Since this vertex is flavor diagonal (in the mass basis), lepton flavor mixing has no effect.

4.2 V-A Operators

4.2.1 Introduction

Without mixing, the $V - A$ operator creates the neutrino pair in a CP even state, thus requiring purely CPV contributions. With mixing the analysis is modified. The effective hamiltonian is modified as follows:

$$\mathcal{H}_{Lead} = \sum_{k,j,\ell} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \lambda_t \widetilde{V}^*_{j\ell} \widetilde{V}_{k\ell} X(x_t, x_\ell)(\bar{s}d)_{V-A}(\overline{\nu}_j \nu_k)_{V-A} + \text{h.c.} \,, \tag{4.3}$$

where [11]:

$$X(x, x_\ell) \cong X_0(x, x_\ell) = \frac{x}{8} \left[\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2} \ln x + O(x_\ell) \right] . \tag{4.4}$$

Neglecting the $O(x_\ell)$ terms in (4.4) leads to the leading order CPV contribution [11]. However, if the $O(x_\ell)$ contribution is not neglected, we find that the outgoing neutrinos do not necessarily carry the same flavor. Since (as seen above) the off diagonal contribution must be proportional to x_ℓ, the electron contribution is negligible. Moreover, in order to get an upper limit on the flavor mixed contributions, the dominance of the internal τ amplitude is assumed which means that:

$$\left| \frac{\widetilde{V}^*_{j\tau} \widetilde{V}_{k\tau}}{\widetilde{V}^*_{j\mu} \widetilde{V}_{k\mu}} \right| > \frac{x_\mu}{x_\tau} \sim \frac{1}{300} . \tag{4.5}$$

Consequently, $\mathcal{H}_{Lead}$ has the following off-diagonal part:

$$\mathcal{H}^{mix}_{Lead} \cong \sum_{k \neq j} \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \lambda_t \widetilde{\lambda}_{jk} \widetilde{X}_0(x_t, x_\tau)(\bar{s}d)_{V-A}(\overline{\nu}_j \nu_k)_{V-A} + \text{h.c.} \,, \tag{4.6}$$

where $\widetilde{X}_0$ is the tau-mass dependent part of $X_0(x, x_\ell)$ in (4.4), and $\widetilde{\lambda}_{jk} \equiv \widetilde{V}^*_{j\tau} \widetilde{V}_{k\tau}$.

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4.2.2 Dirac neutrinos

In the case of Dirac neutrinos the state $|\overline{\nu}_j \nu_k\rangle$ is not a CP eigenstate. Then when $\mathcal{H}^{mix}_{Lead}$ of eq. (4.6) acts on the vacuum it creates the neutrinos in a mixed CP eigenstates. Therefore it contaminates the SM leading decay products, which are generated in a CP even final state. In order to find the magnitude of these new contributions, we calculate the decay rate, summed over all the final mixed flavor states:

$$\Gamma^{mix} \equiv \sum_{j \neq k} \Gamma_{(K_L \to \pi^0 \overline{\nu}_j \nu_k)} = \sum_{j \neq k} \int \left| M(K_L \to \pi^0 \overline{\nu}_j \nu_k) \right|^2 d\Gamma \,, \tag{4.7}$$

where the matrix element M in (4.7) is given by (using standard V_{CKM} parameterization):

$$M(K_L \to \pi^0 \overline{\nu}_j \nu_k) \propto \Im \lambda_t \widetilde{X}_0 \langle \pi^0 | (\bar{s} d)_{V-A} | K^0 \rangle \otimes \langle \overline{\nu}_j \nu_k | \widetilde{\lambda}^*_{kj} (\overline{\nu}_k \nu_j)_{V-A} | 0 \rangle \,. \tag{4.8}$$

Thus the upper bound on the rate is:

$$\Gamma^{mix} \propto \sum_{k \neq j} \left| \Im \lambda_t \widetilde{X}_0 \langle \pi^0 | (\bar{s} d)_{V-A} | K^0 \rangle \, \langle \nu_j \overline{\nu}_k | (\overline{\nu}_j \nu_k)_{V-A} | 0 \rangle \right|^2 \left| \widetilde{\lambda}_{jk} \right|^2 . \tag{4.9}$$

Note that:

$$\sum_{k \neq j} \left| \widetilde{\lambda}_{jk} \right|^2 \leq \frac{3}{4} \,. \tag{4.10}$$

We learn that the new contributions are suppressed by at least $x_\tau^2 = \mathcal{O}(10^{-7})$. The decay is then still dominated by the leading SM CPV contribution to an excellent approximation. Note, however, that the suppression here can be far milder than the one that we found for lepton flavor diagonal decays. Furthermore, the interesting new contributions that lead to a CP odd final state arise from a double insertion of CP violating couplings. Therefore, all mixing related contributions vanish in the CP symmetry limit, similarly to the SM leading CPV contributions.

4.2.3 Majorana neutrinos

For Majorana neutrinos we can rewrite the neutrinos $V-A$ operator as follows [14]:

$$\overline{\nu}_j \gamma^\mu (1 - \gamma^5) \nu_k \propto \nu_j^T \Omega \gamma^\mu (1 - \gamma^5) \nu_k \,, \tag{4.11}$$

where Ω is the charge conjugation matrix. Moreover, for Majorana field $(\nu)^c = \nu$, which leads us to the following CP transformation law for the neutrinos $V-A$ operator:

$$CP \left\{ (\overline{\nu}_j \nu_k)_{V-A} \right\} CP^{-1} \propto (\overline{\nu}_j \nu_k)_{V-A} \,, \tag{4.12}$$

for any j, k. This means that the neutrino pair is created in a CP even state. Therefore the contribution here is singly CPV. The decay rate calculation is the same as in the case of the Dirac neutrinos (eq. (4.7)–(4.9)) and therefore yields the same negligible contribution (4.9) (contributions suppressed by at least x_τ^2). Nevertheless, the suppression here is by far smaller than the one that we found in (3.8).

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4.3 Conclusions

The effects of lepton flavor mixing can be summarized as follows:

(i) The scalar operator CPC contribution is practically unchanged for both Dirac and Majorana neutrinos.

(ii) The V-A operator leads to a new contributions all sensitive to the CPV sector of the theory.

(iii) The ratio between the mixing contribution and the leading CPC contribution without mixing [5] is given by:

$$\sum_{k\neq j}\left(\frac{1}{a_\chi\lambda_c}\frac{M_W^2}{m_K^2}\right)^2\frac{1}{\mathrm{R_{kin}}}\left|\Im\lambda_t\widetilde{\lambda}_{kj}\frac{m_\tau^2}{M_W^2}\right|^2\approx\sum_{k\neq j}\left(\frac{1}{\mathrm{R_{kin}}a_\chi^2}\right)\left|\widetilde{\lambda}_{kj}\frac{\Im\lambda_t}{\lambda_c}\frac{m_\tau^2}{m_K^2}\right|^2$$
$$\approx\sum_{k\neq j}0.1\left|\widetilde{\lambda}_{kj}\right|^2<0.1\,, \qquad (4.13)$$

where $a_\chi=\mathcal{O}(m_K^2/8\pi^2f^2)\sim 0.2$ is a chiral suppression factor, and $\mathrm{R_{kin}}\sim 0.01$ is a phase space integration factor.

Since the mixing-related contribution are at most at the order of the SM CPC contribution, they are negligible and do not change the present status of the $K_L\to\pi^0\nu\bar{\nu}$ decay.

5. Final conclusions

In this work we examined the question of whether the SM CP violating contributions to the $K_L\to\pi^0\nu\bar{\nu}$ decay are still dominant in the presence of three additional types of contributions:

(i) CPC contributions related to massive Dirac neutrinos.

(ii) CPC contributions related to massive Majorana neutrinos.

(iii) Contributions related to flavor mixing effects.

We found an unambiguous answer to our question:

(i) For the massive Dirac neutrinos contribution:

$$\frac{\Gamma_{CPC}^{Dir}(K_L\to\pi^0\nu\bar{\nu})}{\Gamma_{CPV}(K_L\to\pi^0\nu\bar{\nu})}\sim\left(\frac{M_K}{M_W}\frac{m_\nu}{M_W}\right)^2\lesssim 10^{-12}\,. \qquad (5.1)$$

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(ii) For the massive Majorana neutrinos contribution:

$$\frac{\Gamma^{Maj}_{CPC}(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma_{CPV}(K_L \to \pi^0 \nu\bar{\nu})} \sim \left(\frac{\alpha}{64 \sin^2 \Theta_W} \frac{m_t^2}{M_H^2} \frac{m_s^2}{M_W^2} \frac{M_K}{M_W} \frac{m_\nu^{Maj}}{M_W} \right)^2 \lesssim 10^{-29} . \qquad (5.2)$$

(iii) For the flavor mixing effects contribution:

$$\frac{\Gamma^{mix}(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma_{CPV}(K_L \to \pi^0 \nu\bar{\nu})} \sim \sum_{k \neq j} \left| \left(\frac{m_\tau}{M_W} \right)^2 (\widetilde{\lambda}_{jk}) \right|^2 \lesssim 10^{-7} . \qquad (5.3)$$

Moreover, if all neutrinos are lighter than 10 eV, then the m_ν-related contributions are at least 24 (41) orders of magnitude smaller than the CP violating rate in the Dirac (Majorana) case. It is clear then that the $K_L \to \pi^0 \nu\bar{\nu}$ decay process provides a very clean measurement of fundamental, CP violating properties and that it cannot probe neutrino masses.

The above results have interesting implications in the framework of approximate CP. The SM picture of CP violation is not well tested. It could be that ε_K is small because CP is an approximate symmetry and not because of the small SM mixing angles. This would require New Physics to explain ε_K. For example, there exist supersymmetric models with approximate CP [15, 16]. Generally, in such models the CP violating phases fulfill $10^{-3} \lesssim \phi_{CP} \ll 1$, where the lower bound comes from the experimental value of ε_K. Therefore, even if CP violation is accommodated by a source different from the CKM phases, we expect that $\Gamma(K_L \to \pi^0 \nu\bar{\nu})$ is no smaller than three orders of magnitude below the SM rate. Our study implies that even in this extreme case, the CP violating contributions still dominate.

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